Phys 410
Spring 2013
Lecture #6 Summary
4 February, 2013

We started by defining the total momentum $\vec{P}$ of a many particle system as simply the sum over all the particles of the elementary momentum of each particle, $\vec{P} = \sum_{\alpha=1}^{N} \vec{p}_\alpha = \sum_{\alpha=1}^{N} m_\alpha \vec{v}_\alpha$. If the particles in the system interact with each other by means of forces that obey Newton's third law of motion, the change in total momentum is simply the result of a net external force: $\dot{\vec{P}} = \vec{F}_{net}^{ext}$. This is a generalization of Newton's second law of motion to extended systems. An important consequence is that if the net external force is zero, then the total momentum of the many-particle system is conserved.

As an example of momentum conservation of a many-particle system, we considered a rocket in free space, subject to zero net external force. It can begin to move by ejecting mass at a speed v_{ex} relative to the rocket. By conservation of momentum, the rocket gains an equal and opposite moment to that given to the ejected fuel. While describing the momentum of the rocket + exhaust from an inertial reference frame we found that $m\dot{v} = -v_{ex}\dot{m}$, where v is the speed of the rocket, m is its mass, and $\dot{m}$ is the rate at which it is ejecting mass. The thrust force on the rocket is $-v_{ex}\dot{m}$. We also found an expression for the net change in velocity of the rocket as $v - v_0 = v_{ex} \ln \frac{m_0}{m}$, where m_0 is the initial mass and m is the final mass. In order to maximize the rocket velocity one should maximize the exhaust speed v_{ex} and the ratio $\frac{m_0}{m}$. The exhaust speed typically depends on the violent exothermic chemical reaction that takes place in the rocket motor.

We also defined the center of mass of a multi-particle system as $\vec{R} = \frac{1}{M} \sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha$, the weighted sum of the particle positions, where the total mass of the particles is $M = \sum_{\alpha=1}^{N} m_\alpha$. We can relate the total momentum of the system to the center of mass coordinate as $\vec{P} = M\dot{\vec{R}}$. This shows that we can regard the total momentum of the system of particles as if it were a single particle of mass M moving at the velocity of the center of mass. Further, after taking a time derivative we find that $\dot{\vec{P}} = M\ddot{\vec{R}}$ (which assumes that $\dot{M} = 0$), which is Newton's second law for the system of particles in terms of the center of mass momentum derivative and acceleration. This equation justifies our frequent treatment of extended objects (like a baseball, satellite, etc.) as point particles that move on a simple trajectory described by Newton's second law of motion.